

Sol Homework 3 Laplace Transforms : Operations a

1. 0/1 points

ZillDiffEQModAp11M 7.3.003. [3919012]

Find $F(s)$.

$$\mathcal{L}\{t^3 e^{-9t}\}$$

$$F(s) = \boxed{} \times \boxed{\frac{3!}{(s+9)^4}}$$

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$$L\{t^3 e^{-9t}\} = L\{t^3\} \Big|_{s \rightarrow s+9} = \frac{3!}{s^{3+1}} \Big|_{s \rightarrow s+9} = \frac{3!}{s^4} \Big|_{s \rightarrow s+9} = \frac{3!}{(s+9)^4}$$

2. 0/1 points

ZillDiffEQModAp11M 7.3.009. [4571962]

Find $F(s)$.

$$\mathcal{L}\{(1 - e^t + 2e^{-5t})\cos(3t)\}$$

$$F(s) = \boxed{} \times \boxed{\frac{s}{s^2+9} - \frac{s-1}{(s-1)^2+9} + \frac{2(s+5)}{(s+5)^2+9}}$$

$$\begin{aligned} & L\{(1 - e^t + 2e^{-5t})\cos(3t)\} \\ &= L\{\cos(3t) - e^t \cos(3t) + 2e^{-5t} \cos(3t)\} \\ &= L\{\cos(3t)\} - L\{e^t \cos(3t)\} + 2L\{e^{-5t} \cos(3t)\} \\ &= \frac{s}{s^2+3^2} - \frac{s}{s^2+3^2} \Big|_{s \rightarrow s-1} + 2 \cdot \frac{s}{s^2+3^2} \Big|_{s \rightarrow s+5} \\ &= \frac{s}{s^2+3^2} - \frac{(s-1)}{(s-1)^2+3^2} + 2 \cdot \frac{(s+5)}{(s+5)^2+3^2} \\ &= \frac{s}{s^2+9} - \frac{s-1}{(s-1)^2+9} + \frac{2(s+5)}{(s+5)^2+9} \end{aligned}$$

3. + 0/1 points

ZillDiffEQModAp11M 7.3.011. [4571955]

Find $f(t)$.

$$\mathcal{L}^{-1}\left\{\frac{1}{(s+4)^3}\right\}$$

$$f(t) = \boxed{} \times \boxed{\frac{1}{2}e^{-4t}t^2}$$

$$L^{-1}\left\{\frac{1}{(s+4)^3}\right\} = L^{-1}\left\{\frac{1}{s^3}\bigg|_{s \rightarrow s+4}\right\} = \frac{1}{2!}t^2e^{-4t} = \frac{1}{2}t^2e^{-4t} = \frac{1}{2}e^{-4t}t^2$$

Remember :

$$L\{t^n\} = \frac{n!}{s^{n+1}}$$

$$L\{f(t)e^{at}\} = F(s-a)$$

4. + 0/1 points

ZillDiffEQModAp11M 7.3.015. [4572058]

Find $f(t)$.

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2 + 10s + 26}\right\}$$

$$f(t) = \boxed{} \times \boxed{e^{-5t} \cos(t) - 5e^{-5t} \sin(t)}$$

$$L^{-1}\left\{\frac{s}{s^2 + 10s + 26}\right\} = L^{-1}\left\{\frac{s}{(s+5)^2 + 1}\right\} = L^{-1}\left\{\frac{s+5-5}{(s+5)^2 + 1}\right\}$$

$$= L^{-1}\left\{\frac{s+5}{(s+5)^2 + 1}\right\} - L^{-1}\left\{\frac{5}{(s+5)^2 + 1}\right\}$$

$$= L^{-1}\left\{\frac{s}{s^2 + 1}\bigg|_{s \rightarrow s+5}\right\} - 5 \cdot L^{-1}\left\{\frac{1}{s^2 + 1}\bigg|_{s \rightarrow s+5}\right\}$$

$$= e^{-5t} \cos(t) - 5e^{-5t} \sin(t)$$

5. 0/1 points

ZillDiffEQModAp11M 7.3.018. [4572060]

Find $f(t)$.

$$\mathcal{L}^{-1}\left\{\frac{6s}{(s-4)^2}\right\}$$

$$f(t) = \boxed{} \quad \times \quad \boxed{6e^{4t}(4t+1)}$$

$$L^{-1}\left\{\frac{6s}{(s-4)^2}\right\} = L^{-1}\left\{\frac{6(s-4)+24}{(s-4)^2}\right\} = L^{-1}\left\{\frac{6(s-4)}{(s-4)^2}\right\} + L^{-1}\left\{\frac{24}{(s-4)^2}\right\}$$

$$= L^{-1}\left\{\frac{6}{s-4}\right\} + L^{-1}\left\{\frac{24}{(s-4)^2}\right\} = L^{-1}\left\{\frac{6}{s}\right\}_{s \rightarrow s-4} + L^{-1}\left\{\frac{24}{s^2}\right\}_{s \rightarrow s-4}$$

$$= 6 \cdot L^{-1}\left\{\frac{1}{s}\right\}_{s \rightarrow s-4} + 24 \cdot L^{-1}\left\{\frac{1}{s^2}\right\}_{s \rightarrow s-4} = 6 \cdot (1) \cdot e^{4t} + 24 \cdot \left(\frac{1}{1!} \cdot t\right) \cdot e^{4t}$$

$$= 6e^{4t} + 24te^{4t} = 6e^{4t}(1+4t) = 6e^{4t}(4t+1)$$

6. 0/1 points

ZillDiffEQModAp11M 7.3.039. [4572033]

Find $F(s)$.

$$\mathcal{L}\{t \mathcal{U}(t-6)\}$$

$$F(s) = \boxed{} \quad \times \quad \boxed{e^{-6s}\left(\frac{1}{s^2} + \frac{6}{s}\right)}$$

$$L\{f(t-a) \cdot u(t-a)\} = e^{-as} \cdot F(s) \quad \text{if} \quad L\{f(t)\} = F(s)$$

$$L\{t \cdot u(t-6)\} = L\{(t-6+6) \cdot u(t-6)\} = L\{(t-6) \cdot u(t-6) + 6 \cdot u(t-6)\}$$

$$= L\{(t-6) \cdot u(t-6)\} + L\{6 \cdot u(t-6)\}$$

$$= L\{(t-6) \cdot u(t-6)\} + 6 \cdot L\{u(t-6)\}$$

$$= \left(\frac{1!}{s^2}\right) \cdot e^{-6s} + 6 \cdot \left(\frac{1}{s}\right) \cdot e^{-6s}$$

$$= \frac{e^{-6s}}{s^2} + \frac{6e^{-6s}}{s} = e^{-6s}\left(\frac{1}{s^2} + \frac{6}{s}\right)$$

***Notas:**

$$f(t) = t$$

$$L\{f(t)\} = L\{t\} = \frac{1!}{s^2} = \frac{1}{s^2}$$

evidentemente:

$$f(t-6) = (t-6) = t-6$$

7.  0/1 points

ZIII DiffEQ ModAp11M 7.3.041. [4571874]

Find $F(s)$.

$$\mathcal{L}\{\cos(6t) \mathcal{U}(t-\pi)\}$$

$$F(s) = \boxed{} \times \boxed{\frac{e^{-\pi s} s}{s^2 + 36}}$$

$$L\{\cos(6t) \cdot u(t-\pi)\} = L\{\cos(6t-6\pi) \cdot u(t-\pi)\}$$

$$= L\{\cos[6(t-\pi)] \cdot u(t-\pi)\}$$

$$= \frac{s}{s^2 + 6^2} \cdot e^{-\pi s} = \frac{e^{-\pi s} s}{s^2 + 36}$$

***Notas:**

$$f(t) = \cos(6t)$$

$$L\{f(t)\} = L\{\cos(6t)\} = \frac{s}{s^2 + 6^2} = \frac{s}{s^2 + 36}$$

evidentemente:

$$f(t-\pi) = \cos[6(t-\pi)] = \cos(6t-6\pi)$$

para este caso (como situación particular):

$$\cos(6t-6\pi) = \cos(6t)$$

8. 0/3 points

ZillDiffEQModAp11M 7.3.043. [3918278]

Find $f(t)$.

$$\mathcal{L}^{-1}\left\{\frac{e^{-3s}}{s^3}\right\}$$

$$f(t) = \boxed{} \times \boxed{0} + \left(\boxed{} \times \boxed{\frac{1}{2}(t-3)^2} \right) \mathcal{U}(t - \boxed{} \times \boxed{3})$$

$$L^{-1}\left\{\frac{e^{-3s}}{s^3}\right\} = \left(\frac{1}{2!} \cdot t^2 \Big|_{t \rightarrow t-3}\right) u(t-3) = \left[\frac{1}{2}(t-3)^2\right] u(t-3)$$

$$= 0 + \left[\frac{1}{2}(t-3)^2\right] u(t-3)$$

9. 0/1 points

ZillDiffEQModAp11M 7.3.055. [4571914]

Write the function in terms of unit step functions. Find the Laplace transform of the given function.

$$f(t) = \begin{cases} 3, & 0 \leq t < 2 \\ -5, & t \geq 2 \end{cases}$$

$$F(s) = \boxed{} \times \boxed{\frac{3}{s} - \frac{8e^{-2s}}{s}}$$

$$f(t) = 3u(t) - 8u(t-2)$$

$$L\{f(t)\} = L\{3u(t) - 8u(t-2)\}$$

$$= 3L\{u(t)\} - 8L\{u(t-2)\}$$

$$= 3 \cdot \frac{1}{s} - 8 \cdot \frac{e^{-2s}}{s}$$

$$= \frac{3}{s} - \frac{8e^{-2s}}{s}$$

Use the Laplace transform to solve the given initial-value problem.

$$y'' - 17y' + 72y = \mathcal{U}(t - 1), \quad y(0) = 0, \quad y'(0) = 1$$

$$y(t) = \boxed{} \times \boxed{e^{9t} - e^{8t}} + \left(\boxed{} \times \boxed{-\frac{1}{8}e^{8(t-1)} + \frac{1}{9}e^{9(t-1)} + \frac{1}{72}} \right) \mathcal{U}\left(t - \boxed{} \times \boxed{1}\right)$$

$$y'' - 17y' + 72y = u(t-1) \quad y(0) = 0 \quad y'(0) = 1$$

$$L\{y'' - 17y' + 72y\} = L\{u(t-1)\}$$

$$L\{y''\} + L\{-17y'\} + L\{72y\} = L\{u(t-1)\}$$

$$L\{y''\} - 17 \cdot L\{y'\} + 72 \cdot L\{y\} = L\{u(t-1)\}$$

$$\left[s^2 Y(s) - s \cdot y(0) - y'(0)\right] - 17 \cdot [sY(s) - y(0)] + 72Y(s) = \frac{e^{-s}}{s}$$

$$\left[s^2 Y(s) - s \cdot 0 - 1\right] - 17 \cdot [sY(s) - 0] + 72Y(s) = \frac{e^{-s}}{s}$$

$$\left[s^2 Y(s) - 0 - 1\right] - 17 \cdot [sY(s)] + 72Y(s) = \frac{e^{-s}}{s}$$

$$\left[s^2 Y(s) - 1\right] - 17sY(s) + 72Y(s) = \frac{e^{-s}}{s}$$

$$s^2 Y(s) - 1 - 17sY(s) + 72Y(s) = \frac{e^{-s}}{s}$$

$$s^2 Y(s) - 17sY(s) + 72Y(s) - 1 = \frac{e^{-s}}{s}$$

$$s^2 Y(s) - 17sY(s) + 72Y(s) = \frac{e^{-s}}{s} + 1$$

$$(s^2 - 17s + 72)Y(s) = \frac{e^{-s}}{s} + 1$$

$$Y(s) = \frac{\frac{e^{-s}}{s} + 1}{s^2 - 17s + 72}$$

$$Y(s) = \frac{e^{-s}}{s(s^2 - 17s + 72)} + \frac{1}{s^2 - 17s + 72}$$

$$Y(s) = \frac{e^{-s}}{s(s-8)(s-9)} + \frac{1}{(s-8)(s-9)}$$

$$Y(s) = \left[\frac{1}{s(s-8)(s-9)} \right] \cdot e^{-s} + \left[\frac{1}{(s-8)(s-9)} \right]$$

Desarrollando las fracciones parciales con ayuda de Matlab (ver abajo):

$$Y(s) = \left(\frac{A}{s} + \frac{B}{s-8} + \frac{C}{s-9} \right) \cdot e^{-s} + \left(\frac{D}{s-8} + \frac{E}{s-9} \right)$$

$$Y(s) = \left[\frac{\left(\frac{1}{72} \right)}{s} + \frac{\left(-\frac{1}{8} \right)}{s-8} + \frac{\left(\frac{1}{9} \right)}{s-9} \right] \cdot e^{-s} + \left[\frac{(-1)}{s-8} + \frac{(1)}{s-9} \right]$$

$$Y(s) = \left(\frac{1}{72} \cdot \frac{1}{s} - \frac{1}{8} \cdot \frac{1}{s-8} + \frac{1}{9} \cdot \frac{1}{s-9} \right) \cdot e^{-s} - \frac{1}{s-8} + \frac{1}{s-9}$$

$$Y(s) = \frac{1}{72} \cdot \frac{e^{-s}}{s} - \frac{1}{8} \cdot \frac{e^{-s}}{s-8} + \frac{1}{9} \cdot \frac{e^{-s}}{s-9} - \frac{1}{s-8} + \frac{1}{s-9}$$

$$y(t) = L^{-1}\{Y(s)\} = L^{-1}\left\{\frac{1}{72} \cdot \frac{e^{-s}}{s} - \frac{1}{8} \cdot \frac{e^{-s}}{s-8} + \frac{1}{9} \cdot \frac{e^{-s}}{s-9} - \frac{1}{s-8} + \frac{1}{s-9}\right\}$$

$$y(t) = \frac{1}{72} \cdot u(t-1) - \frac{1}{8} \cdot e^{8(t-1)} \cdot u(t-1) + \frac{1}{9} \cdot e^{9(t-1)} \cdot u(t-1) - e^{8t} + e^{9t}$$

$$y(t) = e^{9t} - e^{8t} - \frac{1}{8} \cdot e^{8(t-1)} \cdot u(t-1) + \frac{1}{9} \cdot e^{9(t-1)} \cdot u(t-1) + \frac{1}{72} \cdot u(t-1)$$

$$y(t) = e^{9t} - e^{8t} + \left[-\frac{1}{8}e^{8(t-1)} + \frac{1}{9}e^{9(t-1)} + \frac{1}{72}\right]u(t-1)$$

Comandos de Matlab para obtener fracciones parciales:

Para desarrollar en fracciones $Y(s)$ usando Matlab, en realidad es más fácil usar:

$$Y(s) = \frac{e^{-s}}{s(s^2 - 17s + 72)} + \frac{1}{s^2 - 17s + 72} = \left(\frac{1}{s^3 - 17s^2 + 72s}\right) \cdot e^{-s} + \frac{1}{s^2 - 17s + 72}$$

Con el primer término (en rosa) obtenemos una matriz del numerador $M=[1]$ y una matriz para denominador $N=[1 \ -17 \ 72 \ 0]$. Usando Matlab para desarrollar en fracciones parciales:

```
Command Window
>> format rat
>> M=[1]

M =

     1

>> N=[1 -17 72 0]

N =

     1     -17     72     0

>> [r,p,k] = residue(M,N)

r =

     1/9
    -1/8
     1/72

p =

     9
     8
     0
```

Ello nos permite desarrollar en fracciones parciales la parte de $Y(s)$ en rosa:

$$\left(\frac{1}{s^3 - 17s^2 + 72s} \right) \cdot e^{-s} = \left[\frac{\left(\frac{1}{72} \right)}{s} + \frac{\left(-\frac{1}{8} \right)}{s-8} + \frac{\left(\frac{1}{9} \right)}{s-9} \right] \cdot e^{-s}$$

Nota: observar que el vector $\mathbf{p}$ nos da los polos y el vector $\mathbf{r}$ los residuos (numerador de las fracciones). Deberemos relacionar cada polo con su residuo de acuerdo al orden en que ambos se despliegan en Matlab. Aunque arriba los hemos escrito en otro orden, en relación al orden que arroja Matlab, cada polo de $\mathbf{p}$ está relacionado correctamente con su residuo (numerador en $\mathbf{r}$).

Similarmente, desarrollamos en fracciones parciales el segundo término de $Y(s)$ en azul:

$$\frac{1}{s^2 - 17s + 72}$$

donde nuestra matriz en el numerador es $\mathbf{M}=[1]$ y la del denominador es $\mathbf{N}=[1 \ -17 \ 72]$:

```
Command Window
>> format rat
>> M=[1];
>> N=[1 -17 72];
>> [r,p,k] = residue(M,N)

r =

     1
    -1

p =

     9
     8
```

Relacionando adecuadamente los residuos del vector $\mathbf{r}$ y los polos del vector $\mathbf{p}$:

$$\frac{1}{s^2 - 17s + 72} = \frac{(-1)}{s-8} + \frac{(1)}{s-9}$$